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NONLOCAL ELECTRON HEAT TRANSPORT BY NOT-QUITE MAXWELL-BOLTZMANN DISTRIBUTIONS**

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ABSTRACT

We have made numerical calculations with a new nonlocal fluid treatment of Coulomb collisional electron transport which self-consistently accounts for the nonthermal high energy electrons arising from the spatial transport of thermal electrons whose range is not short compared with the temperature scalelength. Heat fluxes associated with steep gradients are reduced from classical while ahead of a temperature front there is preheating which exceeds classical.

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The need for an efficient self-consistent treatment of high flux electron transport in laser irradiated plasmas has been acute for some time. Intense research has identified and resolved important issues in the Coulomb collisional theory.[1,2] It is now understood that the failure of classical transport occurs when the mean-free-path of the high energy heat-carrying electrons is not much smaller than the temperature scalelength. This implies that the transport fluxes are not locally determined. Indeed, idealizations of systems of interest have been successfully described by a fully multigroup (in energy) diffusion (in space) treatment.[2] This reduced Fokker-Planck theory accounts for the nonthermal distribution induced at high energy by the transport itself; this in turn acts to alter the transport from classical. Heat fluxes associated with steep temperature gradients are reduced from classical, while ahead of the temperature front there is preheating which exceeds classical. This understanding has been gained by analysis of state-of-the-art numerical calculations still too inefficient for application to more than a few problems.

In practice ad hoc fluid schemes have been employed to treat high flux electron transport. The most robust of these limits the heat flux to a fraction, F_{fl}, of its so called free streaming value, $Q_{fl} = F_{fl}n(T/m)^{1/2}T$; however, this prescription is unphysical in that it is pointwise local. Recently a new fluid scheme possessing nonlocal phenomenology has been employed and normalized against certain first principles calculations.[3]

Here we report numerical calculations with a nonlocal fluid treatment of Coulomb collisional transport which self-consistently accounts for the nonthermal high energy electrons arising from the spatial transport of thermal electrons whose range is not short compared to the temperature scalelength. The associated Fokker-Planck calculation has been carried out analytically and permits the first principles formulation of a nonlocal (in space) description of the evolution of the plasma temperature. Our theory requires that the number and energy of the nonthermal electrons be small but their number and energy fluxes are not restricted and may dominate the total fluxes. Indeed, we have recovered in detail the results of previous fully multigroup diffusion laser absorption and heat transport calculations.[2] Our transport scheme is suitable for implementation in one dimensional laser-plasma simulation codes and is expected to be efficient enough for general utility.

The reduced Fokker-Planck equation which governs the electron distibution in position and total energies:[2]

$$\frac{\partial f_0}{\partial t} - \frac{\partial e\phi}{\partial t}\frac{\partial f_0}{\partial \varepsilon} - \frac{1}{v}\frac{\partial}{\partial x}\frac{v^2\lambda_{90}}{6}\frac{\partial f_0}{\partial x} = \frac{df_0}{dt_{ee}} \tag{1}$$

Here $mv^2/2 = \varepsilon + e\phi(x,t)$ and the scattering mean-free-path is $\lambda_{90} = (mv^2)^2/2\pi e^4\ (Z^2N\ \ln\Lambda_{ei} + n\ \ln\Lambda_{ee})$.
Equation (1) neglects the hydrodynamics of the background ions.

The electron number and energy

$$\{n, \frac{3}{2}nT\} = 4\pi \int_{-e\phi}^{\infty} \frac{d\varepsilon v}{m}\{1, \frac{mv^2}{2}\} f_0$$

obey the associated moments of Eq. (1)

$$\frac{\partial}{\partial t}\{n, \frac{3}{2}nT\} + \frac{\partial}{\partial x}\{\Gamma, Q\} + \{0, e\Gamma E\} = 0 \tag{2}$$

Electron-electron collisions, d/dt_{ee}, conserve both number and energy of the species. The electrostatic potential is determined by the requirement of quasineutrality, $n = ZN$. In the present application this becomes the vanishing of the number flux, $\Gamma(e\phi) = 0$. The number and energy fluxes Γ and Q, are

$$\{\Gamma, Q\} = -4\pi \int_{-e\phi}^{\infty} \frac{d\varepsilon}{m}\{1, \frac{mv^2}{2}\}\frac{v^2\lambda_{90}}{6}\frac{\partial f_0}{\partial x} \tag{3}$$

If df_0/dt_{ee} is dominant in Eq. (1) then the distribution takes its local-thermodynamic-equilibrium form, the Maxwell-Boltzmann $f_0 \to f_{MB} = [n/(2\pi\ T/m)^{3/2}]\ \exp[-(\varepsilon + e\phi)/T]$. Employing f_{MB} in Eqs. (3) yields classical transport wherein $Q \propto -\partial T/\partial x$; this closes the fluid Eq. (2).

The transport itself upsets the MB distribution whenever the scale length of interest, L, is not much longer than the stopping length, $\lambda_s = (2\lambda_{90}\ \lambda_\varepsilon/3)^{1/2}$. This may be seen by estimating $df_0/dt_{ee} \sim (v/4\lambda_\varepsilon)f_0$ and comparing it with the spatial diffusion term $\sim (v\lambda_{90}/6L^2)f_0$. Here

$\lambda_\varepsilon = (mv^2)^2/2\pi e^4 n\ell n\Lambda_{ee}$ is the energy loss mean-free-path. On account of the strong energy dependence of λ_s the higher energy electrons are most readily altered by the transport. Inspection of Eq. (3) reveals that these higher energy electrons are the ones which carry the fluxes and so must in turn be accounted for in calculating the transport.

Thus motivated we write $f_0 \simeq f_{MB} + \delta f$ and solve for δf at high energy where Eq. (1) becomes

$$-\frac{1}{v}\frac{\partial}{\partial x}\frac{v^2\lambda_{90}}{6}\frac{\partial f_{MB}}{\partial x} - \frac{1}{v}\frac{\partial}{\partial x}\frac{v^2\lambda_{90}}{6}\frac{\partial \delta f}{\partial x} = \frac{2mv^3}{\lambda_\varepsilon}\frac{\partial \delta f}{\partial \varepsilon} \tag{4}$$

Here f_{MB} is annihilated by d/dt_{ee}, and we have neglected the thermalization of high energy electrons on the bulk as small by $T\partial\ln\delta f/\partial\varepsilon$ compared with the energy loss retained. Also the temporal variation of the distribution and potential are assumed slow. Equation (4) makes explicit that the source (possibly negative!) of nonthermal electrons, δf, is the spatial transport of thermal electrons, f_{MB}.

We solve Eq. (4) in the limit $\varepsilon > -e\phi$ so that $mv^2/2 \sim \varepsilon$ and find

$$f_0 = \int d\xi' \int_\varepsilon^\infty d\varepsilon' \frac{e^{-\frac{(\xi - \xi')^2}{(\varepsilon'^4 - \varepsilon^4)}}}{[\pi(\varepsilon'^4 - \varepsilon^4)]^{1/2}} \frac{f_{MB}(\xi', \varepsilon')}{T(\xi')} \tag{5}$$

Here $d\xi = dx/\tilde{\lambda}_s$ and we use $\tilde{\lambda} = \lambda/(mv^2)^2$. We have not yet taken account of spatial boundary conditions. We have also passed a factor of $Z^{1/2}$ through $\partial/\partial x$ in defining ξ and thus require that the ionization state vary more slowly than the temperature.

Equation (5) shows that nonthermal electrons at ξ and ε have come from all ξ' and all $\varepsilon' > \varepsilon$ by stopping transport. We observe that f_0 of Eq. (5) is determined by the density and temperature of f_{MB}. We shall neglect the small number and energy, δn and $\delta 3nT/2$, of nonthermals in the fluid Eqs. (2) so that the density and temperature of the plasma become those of the MB distribution. Thus substitution of f_0 of Eq. (5) into Eqs. (3) closes the fluid Eqs. (2) and naturally yields a nonlocal treatment of the transport.

In Eqs. (3) we have energy integrals of $\partial f_0/\partial x = (1/\tilde{\lambda}_s)\ \partial f_0/\partial \xi$; this differentiation acts on the Gaussian kernel in Eq. (5) for f_0. We replace $\partial/\partial\xi$ by $-\partial/\partial\xi'$ and integrate by parts in ξ' to cast the spatial derivative onto f_{MB}/T. The electric field enters explicitly upon differentiation of the exponential factor of f_{MB}. It is convenient to split the potential into local and nonlocal parts, $e\phi = e\phi_l + e\phi_{nl}$, where $\partial e\phi\ell/\partial\xi = (T/n)\ \partial n/\partial\xi - (5/2)\ \partial T/\partial\xi$. Finally $\epsilon > -e\phi$ is exploited by letting $-e\phi \to 0$ everywhere except under differentiation by ξ'.

The resulting double energy integrals have been computed to obtain

$$\{\Gamma,\ Q\} = -\frac{(\tilde{\lambda}_{90}/\tilde{\lambda}_\epsilon)^{1/2}}{4\pi\ (3m)^{1/2}} \int dx'\ n\ T^{-1/2}\ \{1,\ T\}\ [\frac{\partial T}{\partial x'}\{I,\ K\} - \frac{\partial e\phi}{\partial x'}\ \{J,\ L\}] \tag{6}$$

Here we have made the transformation $d\xi'\partial/\partial\xi' = dx'\partial/\partial x'$. The nonlocal transport propagators P = I, J, K and L are functions only.

$$\theta = \int_x^{x'} \frac{dx''}{\tilde{\lambda}_s} \ /\ T^2(x')$$

which is the number of stopping lengths from the source point x' to the field point x at an energy equal to the source temperature.

The propagators have the following form

$$P(\theta) = {}^2\theta + 2\beta \int_0^\infty dy\ y^\beta\ e^{[-\theta^{1/2}y^{1/2}-1/y]} \int_0^1 dy'\ \frac{y'^\alpha}{(1-y')^{1/2}}\ e^{[y'/y(1-y')]}$$

For small θ they fall off linearly, $P(\theta) \to P(0) + P'(0)\theta$, while for large θ they fall off exponentially in $\theta^{2/5}$, $P(\theta) \to 32\ (2\pi/5)^{1/2}\ \theta^\gamma\ \exp[-(5/4^{4/5})\ \theta^{2/5}]$. The constants α, β, P(0), P'(0), A, γ and the propagator normalization integrals are shown in Table I.

We have analysed two limiting temperature profiles which illuminate the physics in play. First, in the limit of gradients much longer than the stopping length. The nonlocal heat flux reduces to classical because only x' near x contributes in the integrals of Eq. (6). We let $dx' \to T^2\tilde{\lambda}_s d\theta$ and employ the propagator normalizations. Taking $\partial e\phi_{nl}/\partial x = 5\ \partial T/\partial x$ from $\Gamma = 0$ yields the classical results; $\partial e\phi/\partial x \to (T/n)\ \partial n/\partial x + (5/2)\ \partial T/\partial x$ and $Q \to 25.532\ n\ (T/m)^{1/2}$ $\lambda_{mfp}\ \partial T/\partial x$. Here $\lambda_{mfp} = T^2\ \tilde{\lambda}_{90}$.

Second our nonlocal heat flux is naturally self-limiting! For a temperature step from Hot to Cold over a distance much shorter than a stopping length we find; $\Delta e\phi_{nl} \to 3\ \Delta T$ and $Q \to 1.285\ (\tilde{\lambda}_{90}/\tilde{\lambda}_{\varepsilon})^{1/2}$ $(n/m^{1/2})\ (T_H^{\ 3/2} - T_C^{\ 3/2})$. Note that the temperature within the transition interval does not enter here; the limit of our nonlocal treatment is indeed nonlocal.

We have also solved the fluid Eqs. (2) numerically. Figure 1 shows some results of illuminating a stationary plasma with a constant intensity laser beam. To this end we have introduced a source κI in the energy equation.[4] The calculation was terminated when the heat flux into the overdense plasma equaled the absorbed laser flux so that the coronal plasma had reached a quasisteady condition.

In Fig. 1 we note the signatures of the nonlocal transport are 1) a hot absorption layer, 2) a steep temperature gradient separating the laser heated plasma and conduction heated plasma, 3) a non-isothermal corona and, 4) a preheat foot on the temperature profile. The reduced absorption recorded in the table is associated with 1) since $\kappa_C \propto T^{-3/2}$; this feature and 2) are typical of strongly flux limited simulations required to reproduce results of laboratory experiments. Here 1) and 2) are due to the stopping of electrons as they transport from the laser heated plasma. The non-isothermal corona results from nonlocal transport from there towards the colder denser plasma. This physics is also responsible for the preheat foot.

Figure 2 shows that including nonlinear reduction of the laser opacity[5] brings the absorbed energy, heat flux and temperature all into agreement with previous direct solutions of Eq. (1).[2] Indeed, at high energy the underlying nonlocally determined distribution function, f_0 of Eq. (5), is in agreement as well. In our transport treatment we have neglected the effect of the nonthermal distribution induced at low energy by laser heating.[5] This heating is communicated to the high energy transporting electrons via the temperature T; self collisions mediate between laser heating at low energy and transport at high energy.

The effect of the nonlocally determined electric field is displayed in Fig. 2. The field acts to reduce the heat flow from that due to the temperature gradient. The reduction is substantial and is not constant, being both larger and smaller than in the classical limit. In particular,less than classical reduction occurs in the preheat region because only a relatively small electric field is required to establish zero total current against the small current of high energy heat transporting electrons.

We have checked that neglect of thermalization of high energy electrons on the bulk electrons is justified.[6] Only modest errors of our nonlocal treatment are indicated.

Energy conservation in the laser heated plasma calculations requires that there be no heat flux into the vacuum. This implies the condition $\partial f_0/\partial x = 0$ at the boundary; this is straightforward but tedious to implement as an (infinite) series of images. Provided the entire system is many stopping lengths in extent it is sufficient to include a single image plasma on the vacuum side of the fluid calculation. In our calculations we extended the Γ and Q integrations over this image plasma to ensure that the fluxes vanished correctly at the boundary.

References

1. A. R. Bell, R. G. Evans and D. J. Nicholas, Phys. Rev. Lett. 46, 243 (1981); R. J. Mason, Phys. Lett. 47, 652 (1981); J. P. Matte and J. Virmont, Phys. Rev. Lett. 49, 1936 (1982).

2. J. R. Albritton, Phys. Rev. Lett. 50, 2078 (1983) and Lawrence Livermore National Laboratory Report UCRL-87931, 1983 (unpublished).

3. J. F. Luciani, P. Mora, and J. Virmont, Phys. Rev. Lett. 51, 1664 (1983).

4. The laser intensity obeys $dI/dx = -\kappa I$; the laser opacity is $\kappa = K(I,n,T)\kappa_c$. Here K is the reduction factor given by Langdon[5] and κ_c is the classical opacity.

5. A. B. Langdon, Phys. Rev. Lett. 44, 575 (1980).

6. J. R. Albritton, E. A. Williams, I. B. Bernstein and K. P. Swartz, Lawrence Livermore National Laboratory Laser Program Annual Report UCRL-50021-85, 1986 (unpublished).

Figure Captions

1. Nonlocal transport gives significantly different results from classical in a calculation of a laser heated plasma; classical absorption, K = 1.0.[4]

2. Previous fully multigroup diffusion transport results[2] are recovered by the nonlocal treatment when nonlinear reduction of the laser opacity[4,5] is introduced; here K ~ 0.7.

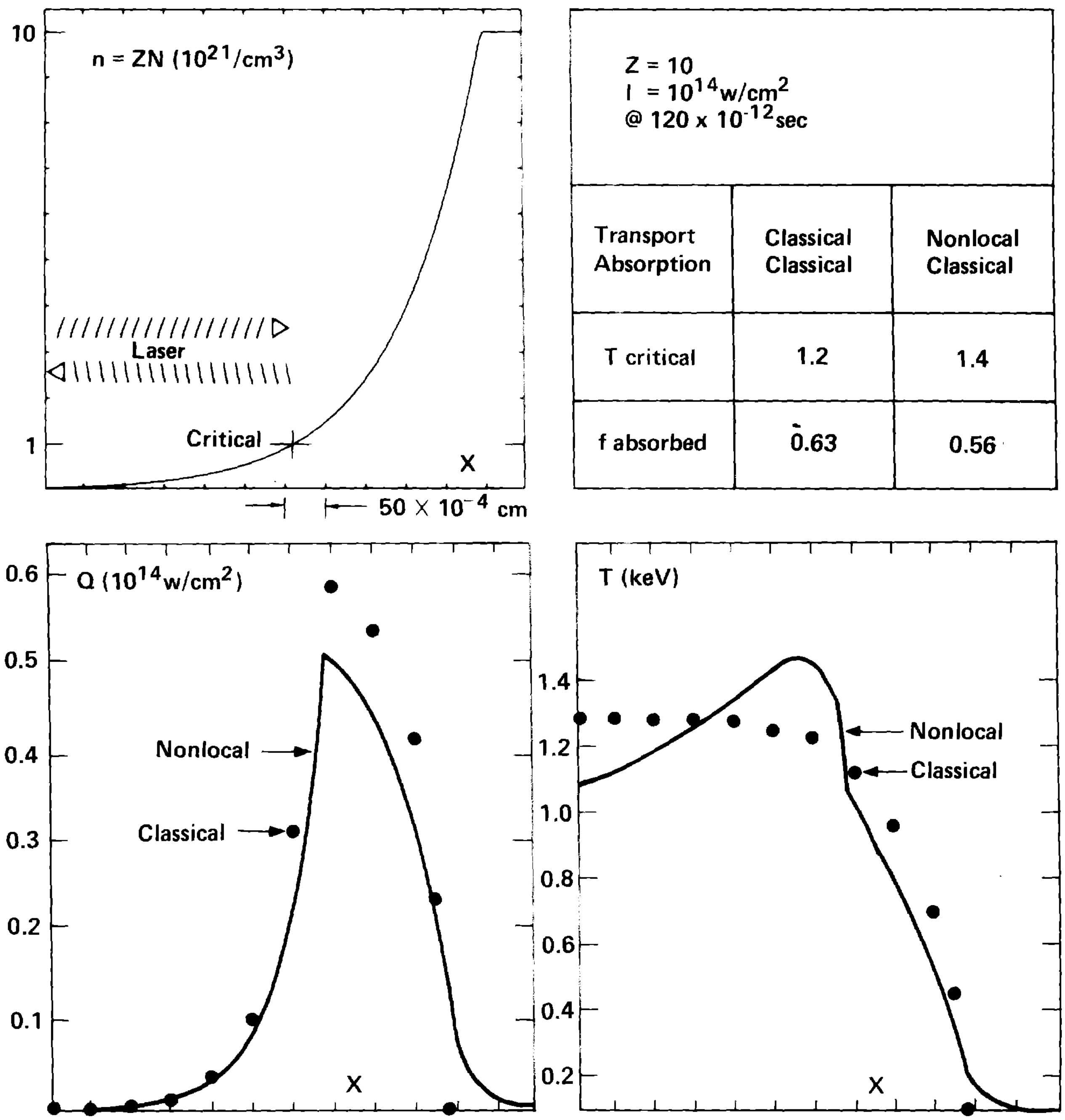
Table Caption

1. Coefficients for the nonlocal transport propagators described in the text.

P	α	β	P(0)	-P'(0)	A	γ	$\int_0^\infty d\theta P(\theta)$
I	0	0	48	$8\sqrt{\pi}$	1	1	$240\sqrt{\pi}$
J	0	$-\frac{1}{4}$	16	$8\sqrt{\pi}$	$4^{-\frac{1}{5}}$	$\frac{3}{5}$	$48\sqrt{\pi}$
K	$\frac{1}{4}$	$\frac{1}{4}$	$\dfrac{24\sqrt{\pi}\,\Gamma(\frac{5}{4})}{\Gamma(\frac{7}{4})}$	$16\sqrt{\pi}$	$4^{\frac{2}{5}}\,\Gamma\,(\frac{5}{4})$	$\frac{13}{10}$	$1152\sqrt{\pi}$
L	$\frac{1}{4}$	0	$\dfrac{96\sqrt{\pi}\,\Gamma(\frac{5}{4})}{\Gamma(\frac{7}{4})}$	$8\sqrt{\pi}$	$4^{\frac{1}{5}}\,\Gamma\,(\frac{5}{4})$	$\frac{9}{10}$	$192\sqrt{\pi}$

Table I

0158p

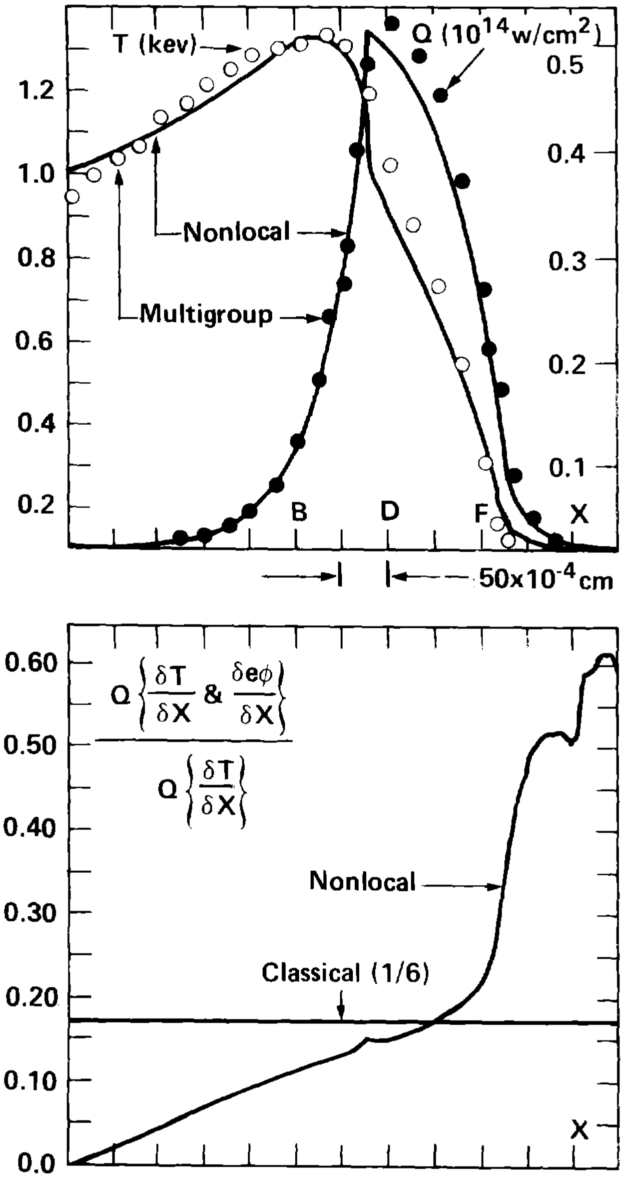


Transport Absorption	Classical Classical	Nonlocal Classical
T critical	1.2	1.4
f absorbed	0.63	0.56

Figure 1

$Z = 10$
$I = 10^{14}$ w/cm^2
@ 120×10^{-12} sec

Transport Absorption	Nonlocal Nonlinear	Multigroup Multigroup
T critical	1.3	1.3
f absorbed	0.47	0.46



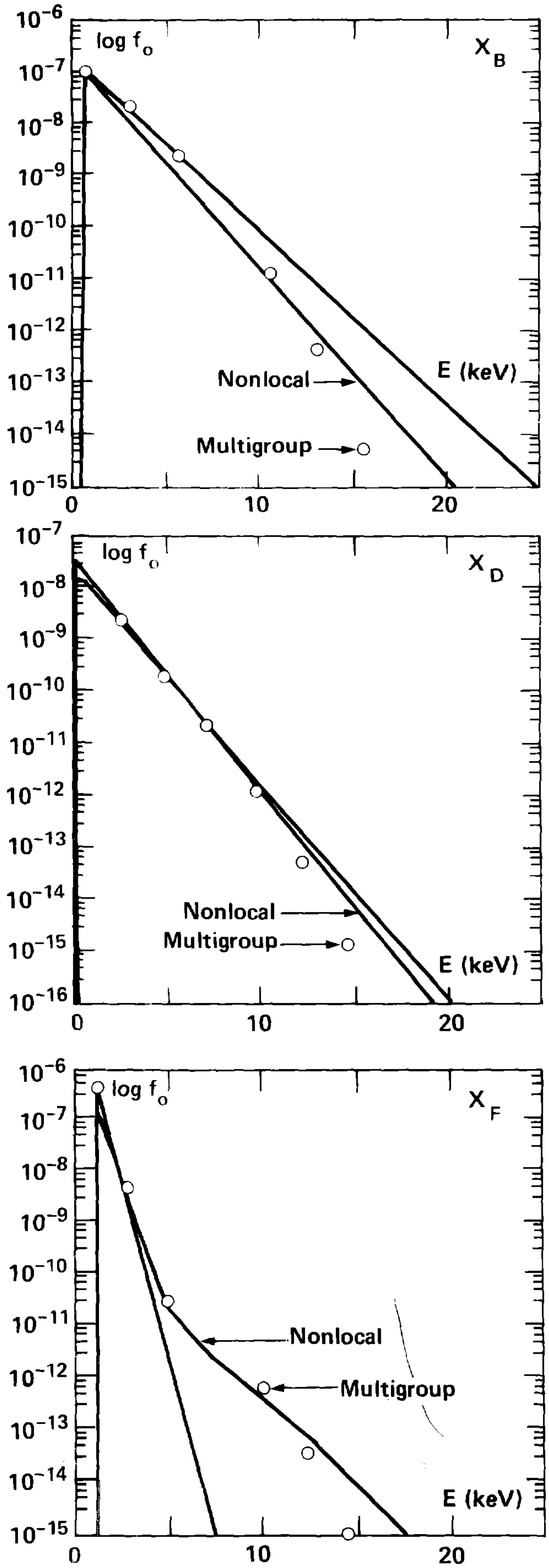


Figure 2